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Blind Separation of Twin Fetal Heartbeats in an Electrocardiogram using the Fractional Fourier Transform

Seema Sud
(Communications and Signal Analysis Department, The Aerospace Corporation, USA)

ABSTRACT
The Fractional Fourier Transform (FrFT) has been used for signal separation in numerous applications, such as speech, radar, and image processing. It is a useful signal processing tool that enables significantly greater signal separation by rotating signals in a time-frequency plane known as the Wigner Distribution (WD) where they may be totally separable, when they are not separable in the time or frequency domains alone. In this paper, we apply the FrFT to the problem of separating the heartbeats of twin fetuses, as seen on an electrocardiogram (ECG), from each other. Most techniques fail here because the two heartbeat patterns are similar. The proposed algorithm takes advantage of the fact that two or more fetal heartbeats can be approximated as damped, chirp signals and may be slightly offset in time or amplitude. Thus, the WDs of the two signals will be slightly different from each other. By finding and rotating to the proper axis in the WD plane, where the chirp heartbeats become narrowband, separable signals, we can notch much of the weaker heartbeat to extract the stronger one. The estimate of the stronger signal is then subtracted from the ECG signal to obtain the weaker one. We show that the technique operates well using simulations, even when the two heartbeats are nearly equal in power. Specifically, we show that the mean-square error (MSE) between the true heartbeats and the estimated ones are only on the order of 10^{-4}–10^{-3}. This method will enable identification of problems in an unborn child early in when the pregnant mother is expecting twins.
Keywords – Biomedical, Chirp, Electrocardiogram, Fractional Fourier Transform, Heartbeat, Twins

I. INTRODUCTION

Detecting problems in the unborn fetus of a pregnant woman early is an important area of research in the medical field. This can be done by collecting the signal in an electrocardiogram (ECG) and studying the nature of the fetal heartbeat, i.e. pulse strength and frequency. Separation of the fetal heartbeat from that of the pregnant mother can be done by adaptive filtering methods. For example, an electrode placed on the heart of the expectant mother will provide a reference signal for filtering her heartbeat out from the combined (i.e. mother and fetus) signal collected from the electrode placed on her abdomen [1]. Such techniques also use the fact that the fetal heart rate is often much faster and weaker than that of the mother, thereby enabling easy separation [1]. Separation of noise from the ECG signal can also be achieved with adaptive filtering ([2] and [3]). However, when a pregnant mother-to-be is expecting two or more babies, separating the fetal heartbeats is a problem. This is due to the fact that the two heartbeats overlap in time and frequency, and are typically very similar in their spectral features. If the heart rates of the two fetuses are similar, the problem becomes even more challenging.

In this paper, we propose to use the Fractional Fourier Transform (FrFT) to separate the heart signals from two fetuses in an ECG. The FrFT is most suitable for this problem because the fetal

Heartbeats will often have slight time offsets, as well as repetition rate offsets, making them more separable in the FrFT domain than in the conventional frequency (i.e. FFT) domain. Specifically, if we upsample the ECG signal, then the heartbeats will resemble chirp functions, each one with a different time-frequency representation [4]. Using small amplitude differences, detection of the FrFT rotational axis a_{opt} which produces the maximum peak in the stronger heartbeat, followed by rotation to this optimum FrFT axis, t_{aopt}, will allow us to notch out the weaker signal and estimate the stronger one. Subtraction of the stronger one from the composite signal gives a good estimate of the weaker heartbeat.

The paper outline is as follows: Section II briefly reviews how to compute the FrFT of a signal. Section III describes the signal model. Section IV discusses the proposed algorithm for separation of two or more ECG signals using the FrFT. Section V presents simulation examples showing the robust performance of the proposed FrFT method. Conclusions and remarks on future work are given in Section VI.

II. BACKGROUND: FRACTIONAL FOURIER TRANSFORM (FRFT)

In discrete time, we can model the N ×1 FrFT of an N ×1 vector **x** as

$$\mathbf{X}_a = \mathbf{F}^a \mathbf{x}, \quad (1)$$

where 0<|a|<2, $\mathbf{F}^a$ is an N×N matrix whose elements are given by ([5] and [6])

$$\mathbf{F}^a[m,n] = \sum_{k=0, k\neq (N-1+(N)_2)}^{N} u_k[m] e^{-j\frac{\pi}{2}ka} u_k[n], \quad (2)$$

$u_k[m]$ and $u_k[n]$ are the eigenvectors of the matrix **S** [5]

$$\mathbf{S} = \begin{bmatrix} C_0 & 1 & 0 & \dots & 1 \\ 1 & C_1 & 1 & \dots & 0 \\ 0 & 1 & C_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & C_{N-1} \end{bmatrix}, \quad (3)$$

and

$$C_n = 2cos(\frac{2\pi}{N}n) - 4. \quad (4)$$

Hence, the discrete time FrFT can be implemented using a simple matrix-vector multiplication. Many references are available on the properties of the FrFT as well as its relation to the WD, e.g. [6], so for brevity we omit details.

III. ECG SIGNAL MODEL

We assume that the mother's heartbeat has already been separated from that of the unborn babies, as there are simple ways of doing this using adaptive filtering methods in which electrodes strategically placed near the mother's heart can be used to obtain a clean reference signal, enabling her heartbeat signal to be estimated and subtracted out [1]. Additional distortions, however, may be present, due to the mother's breathing, mother's muscle contractions, or other artifacts that cannot be perfectly subtracted, so we include this as noise in the model below. The ECG signal for the first fetal heartbeat, $x_1(i+1)$ is modeled by computing, for i = 1,2,...,L and ∀l = 1, 2, ..., 14

$$i = d(l) : d(l+1) - 1, \quad (5)$$

$$m = \frac{a(l+1) - a(l)}{d(l+1) - d(l)}, \quad (6)$$

$$x_1(i+1) = a(l) + m \times \{i - d(l)\}, \quad (7)$$

$$d = round(d_0 \times L/d_0(15)), \quad (8)$$

Where L = 1,600, d(15) = L, a = a_0/max(a_0), and d_0 and a_0 are constant 1×15 vectors. This function produces a piecewise linear signal that we upsample and filter using a Savitzky-Golay filter [7] with a frame size F = 21[1]. Another fetal ECG signal, $x_2(i)$ is similarly generated, scaled by amplitude A_2, delayed by L_τ samples, upsampled, and then filtered. Including the presence of an additive white Gaussian noise (AWGN) signal, n(i), we can write the composite received signal as

$$y(i) = x_1(i) + A_2 x_2(i - L_\tau) + n(i), \quad (9)$$

Where n(i) is chosen for a given signal-to-noise ratio (SNR). In vector form, we can write all L samples as

$$\mathbf{y} = \mathbf{x}_1 + A_2 \mathbf{x}_2 + \mathbf{n}. \quad (10)$$

The goal is to separate signals 1 and 2 from the received signal **y**. We discuss the proposed approach in the following section, but we point out here that this is a more difficult problem than found in communications or radar, for example, because (1) we do not have any training information, and (2) the two signals to be separated are very similar in nature, and will be more stationary than other types of signals, making signal separation more difficult.

IV. PROPOSED ECG SIGNAL SEPARATION METHOD USING THE FRFT

We propose to separate the two fetal heartbeats using the Fractional Fourier Transform (FrFT). The proposed algorithm uses the fact that the two heartbeats may have slightly different amplitudes and time offsets from one another, making them more separable in the WD plane than in the FFT domain. We obtain an estimate of the slightly stronger heartbeat in the domain where the two signals are best separated by searching over all 'a', finding the value of 'a' where the signal peak is strongest, and rotating to the best value of 'a' called a_{opt}, as illustrated in Fig. 1. This estimate of a_{opt} is corrupted by the second signal's interference, but we have no a priori knowledge to obtain a better estimate. In this domain, we notch out the weaker signal by finding the peak bin i_{opt} of signal 1 and then estimating the number of bins about i_{opt} occupied mostly by signal 1, i.e. $i_{opt}±\Delta i$. The estimate of Δi is obtained by observation of the spectrum at a_{opt}. Following notching of the weaker signal in the FrFT domain, we then rotate back to the time domain. Note that we cannot apply the same method for the weaker heartbeat, as the FrFT peak from searching over all 'a' only reveals the stronger heartbeat. However, simply subtracting the signal 1 estimate from the received signal gives us an estimate of the second heartbeat. Hence, only one

[1] These functions are modeled using ecg.m and sgolayfilt.m in MatLab's signal processing toolbox (©MatLab 1988 – 2004).

FrFT search over ‘a’ is required. The procedure is provided below, where we let the step size be Δa = 0.001 and Δi = 10 samples (FrFT domain ‘frequency’ bins). Note that we use $\mathbf{Y}_1$ to denote the entire signal in vector form, but we use $Y_1(i)$ to denote the individual samples at index i for the signal, for ease in describing the procedure.

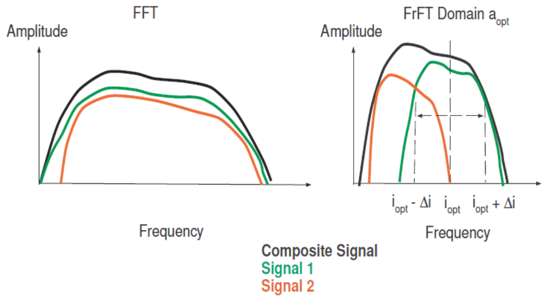

Fig. 1 Comparison of FFT vs. FrFT domain shows signal separation ability with FrFT in domain $\mathbf{a_{opt}}$

For 0≤a<2, we compute the FrFT of **y** as

$$\mathbf{Y}(a) = \mathbf{F}^{a}\mathbf{y}. \quad (11)$$

We select the maximum value by computing

$$Y_{max}(a) = max(|\mathbf{Y}(a)|). \quad (12)$$

Next, we find peak over all ‘a’ to give the best domain to find the stronger of the two signals

$$a_{opt} = \underset{a}{arg\ max}\ Y_{max}(a). \quad (13)$$

Now we rotate to this best ‘a’, where the stronger signal energy is most concentrated

$$\mathbf{Y}_1 = \mathbf{F}^{a_{opt}}\mathbf{y}. \quad (14)$$

We find the sample where the peak occurs

$$i_{opt} = \underset{i}{arg\ max}\ Y_1(i) \quad (15)$$

and notch out signal 2[2]

$$Y_1(0 : i_{opt} - \Delta i, i_{opt} + \Delta i : L) = 0. \quad (16)$$

We rotate back to the time domain to get the estimate of signal 1

$$\hat{\mathbf{x}}_1 = \mathbf{F}^{-a_{opt}}\mathbf{Y}_1. \quad (17)$$

Finally, we compute the estimate for signal 2 simply by taking the received signal and subtracting the estimate of signal 1

$$\hat{\mathbf{x}}_2 = \mathbf{y} - \hat{\mathbf{x}}_1. \quad (18)$$

To determine how well the algorithm performs, we compute the mean-square error (MSE) between the true signal k, k = 1 or 2, and its estimate using

$$MSE_k = \sqrt{\sum_{i=1}^{L}\{x_k(i) - \hat{x}_k(i)\}^2 / L}. \quad (19)$$

Note that we can perform all of the above calculations using a single pulse. Hence, the algorithm can operate in real-time with very few samples. Note also that there may in practice be an ambiguity in knowing which heartbeat belongs to which fetus. This can be addressed by subsequent measurements of the composite ECG signal, using knowledge of the estimates with their power and time offsets. Finally, note that we cannot obtain perfect signal separation, due to the signals overlapping, even in the FrFT domain; the error resulting from this overlap will be seen in the examples below. This would improve if the heart rates of the two fetuses are different, but we assume here that they are the same and still achieve good performance.

V. PERFORMANCE RESULTS

We simulate two signals $x_1(i)$ and $x_2(i)$ in noise, where we vary the amplitude of the second signal, A_2 to study detection performance. We generate the composite received signal **y** by sampling, scaling, and delaying two ECG signals in MatLab, which produces a single snapshot in time of the two fetal heartbeats. With SNR = 10 dB, A_2 = 0.9 and L_τ = 600, we repeat the signal N = 5 times and add noise to generate a longer signal, as shown in Fig. 2. For processing using the above algorithm, we only need a single snapshot, so, we restrict our attention to the case where N = 1.

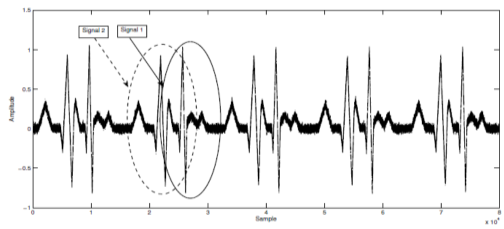

Fig. 2 Received Signal; SNR = 10 dB, $\mathbf{A_2}$ = 0.9, $\mathbf{L_\tau}$ = 600 samples, and N = 5 Repetitions

In the first example, we let the SNR be 10 dB and set A_2 to 0.9, so that the carrier-to-interference ratio (CIR) between signals 1 and 2 is 1 dB. We let the delay between signals 1 and 2 be 600 samples out of the total L = 1,600 samples. The received signal, the recovered signals, and the true signals are shown in Fig. 3. The MSEs are $MSE_1 = 8.6 \times 10^{-4}$ and $MSE_2 = 8.4 \times 10^{-4}$. Note from Fig. 1 that since we cannot separate the signals in the FFT domain, we cannot compare performance of any FFT-based algorithm to the proposed method. Attempts to use the FFT for this problem would fail.

[2] Note that we could instead notch signal 1 using $Y_1(i_{opt}-\Delta i : i_{opt} + \Delta i) = 0$.

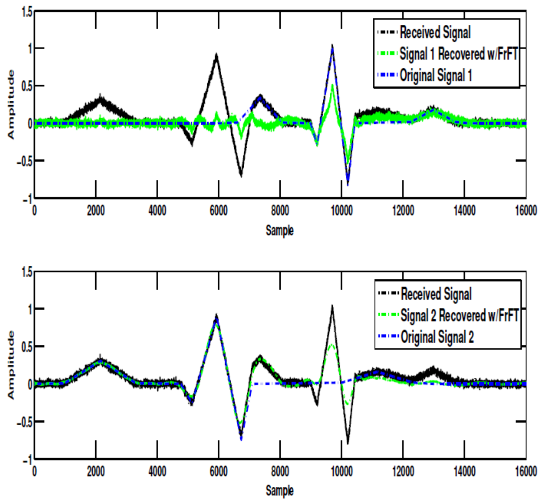

Fig. 3 SNR = 10 dB, A_2 = 0.9, L_τ = 600 samples, $MSE_1 = 8.6 \times 10^{-4}$, and $MSE_2 = 8.4 \times 10^{-4}$

In the second example, we let $A_2 = 0.5$, so that now CIR = 6 dB, with the results shown in Fig. 4. The MSEs are now $MSE_1 = 8.2 \times 10^{-4}$ and $MSE_2 = 8.0 \times 10^{-4}$.

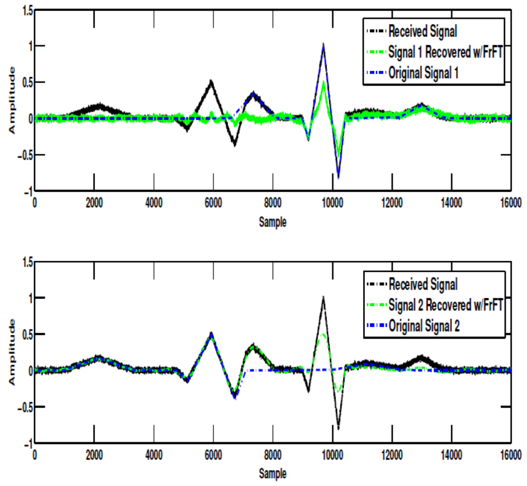

Fig. 4 SNR = 10 dB, A_2 = 0.5, L_τ = 600 samples, $MSE_1 = 8.2 \times 10^{-4}$, and $MSE_2 = 8.0 \times 10^{-4}$

The third example uses $A_2 = 0.9$ as in the first example, but now signal 2 is delayed 900 samples with respect to signal 1. The results are shown in Fig. 5. The MSEs are now $MSE_1 = 7.6 \times 10^{-4}$ and $MSE_2 = 7.3 \times 10^{-4}$. Note that there is less of an overlap of the two signals in time, and hence the performance improves slightly.

The final example uses $A_2 = 0.5$ and a delay of signal 2 of 900 samples with respect to signal 1. Hence there is again less of an overlap in time and a greater power separation, so we expect performance improvement, with the results shown in Fig. 6. The MSEs are now $MSE_1 = 4.6 \times 10^{-4}$ and $MSE_2 = 4.3 \times 10^{-4}$. These results, summarized in Table 1 below, show that a greater amplitude difference in the two signals and a greater time offset improves the performance of the algorithm.

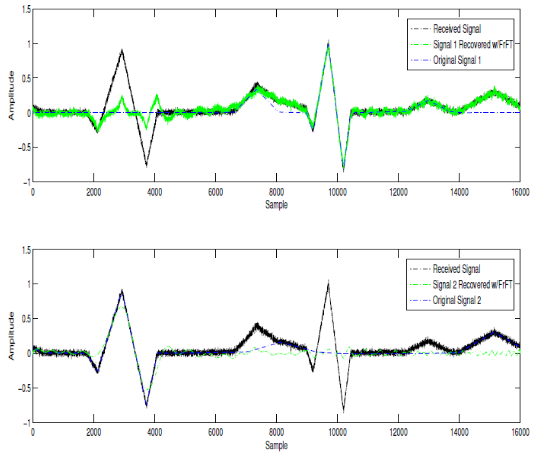

Fig. 5 SNR = 10 dB, A_2 = 0.9, L_τ = 900 samples, $MSE_1 = 7.6 \times 10^{-4}$, and $MSE_2 = 7.3 \times 10^{-4}$

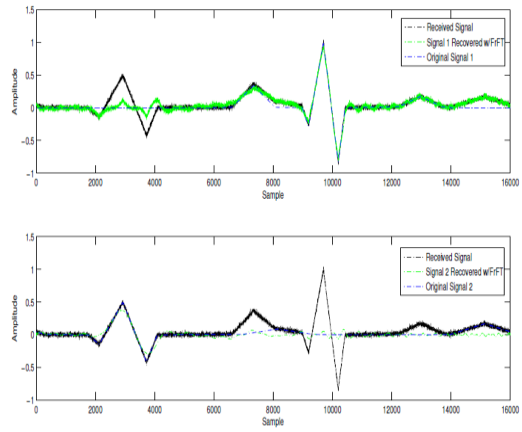

Fig. 6 SNR = 10 dB, A_2 = 0.5, L_τ = 900 samples, $MSE_1 = 4.6 \times 10^{-4}$, and $MSE_2 = 4.3 \times 10^{-4}$

Table 1 MSE results summary

Example	A_2	Delay L_τ	MSE_1	MSE_2
1	0.9	600	8.6×10^{-4}	8.4×10^{-4}
2	0.5	600	8.2×10^{-4}	8.0×10^{-4}
3	0.9	900	7.6×10^{-4}	7.3×10^{-4}
4	0.5	900	4.6×10^{-4}	4.3×10^{-4}

VI. CONCLUSION

In this paper, we apply the Fractional Fourier Transform (FrFT) to separating the heartbeats of twin fetuses in a pregnant woman. This problem is difficult because the heartbeats have very similar features. We show that we are able to separate the signals when there are slight power and time offsets between them, which gives them different Wigner Distributions, thereby enabling separation. Future work involves improving the performance of the algorithm and developing new algorithms for the case where there are three or more fetuses.

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